A Fuzzy, Non-Linear Similarity Measure for a Clinical Case-based Reasoning System

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Abstract. This paper presents a fuzzy, non-linear similarity measure designed for a clinical case-based reasoning system in radiotherapy treatment planning. The developed fuzzy similarity measure takes into account the distribution of attribute similarity values in the case base to ensure that the numerical values of the similarity between individual attributes are comparable and can be combined to give the aggregate similarity between two cases. Local fuzzy membership functions that are based on the attribute values of the target case are defined. The performance of the fuzzy similarity measure using local fuzzy membership functions is evaluated using real world brain cancer patient data. Preliminary experiments show promising results.

Keywords: Fuzzy Similarity Measure, Local Fuzzy Membership Functions, Clinical Decision Support Systems, Radiotherapy Treatment Planning

1 Introduction

Case-based Reasoning (CBR) is based on the premise that similar cases have similar solutions and the similarity between two cases is an indication of how applicable the solution of a case is to the target case. Therefore, in order to retrieve cases with suitable solutions, the choice and design of the similarity measure is an important consideration when creating a CBR system. The total similarity between two cases is often computed as the weighted sum of the individual attribute similarity values. However, in order to compute the total similarity, this method assumes that the similarity values between attributes are comparable. However, if the distribution of similarity values with respect to different attributes is different, the numerical attribute similarity values might not be directly comparable. In this paper, we question the assumption that a similarity value S_A with respect to attribute A always denotes the same extent of similarity as similarity value S_B with respect to attribute B, if $S_A = S_B$. Normalisation of attribute values only accounts for the scale or range of attribute values and not for the variability of the similarity in between the extreme values of small and large attribute similarity between two cases. In this work, we propose the use of a fuzzy, non-linear similarity measure that takes into account the distribution of attribute similarity values in a CBR system for radiotherapy treatment planning.

Radiotherapy is a form of cancer treatment in which tumour cells are destroyed by subjecting them to ionising radiation. However, since excessive radiation adversely affects all cells, including healthy tissue and critical organs, a detailed treatment plan is required for each patient that describes exactly how a patient is irradiated in order to deliver a tumouricidal radiation dose over the tumour region while minimising the radiation received by healthy tissue and critical organs in the vicinity of the tumour. Oncologists use their subjective experience and expert clinical knowledge to generate the plan parameters using a trial and error approach that can take from a few hours to several days in complicated cases. The developed prototype system for radiotherapy planning uses case-based reasoning to capture this kind of intuitive and

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empirical knowledge to aid oncologists and medical physicists in the computation of plan parameters. The system suggests possible treatment plan parameters for a new patient based on the treatment plans of previously successfully treated similar patient cases.

The CBR system is developed in collaboration with the City Hospital, Nottingham University Hospitals NHS Trust, UK. Real-world data of brain cancer patients are used in the experiments.

The paper is organized as follows. Section 2 describes the CBR system under development for radiotherapy treatment planning for brain cancer. Section 3 briefly discusses the motivation behind the design of the non-linear, fuzzy similarity measure. The fuzzy similarity measure, including global and local fuzzy membership functions, is presented in section 4. Section 5 presents experimental results obtained with labeled data to evaluate the performance of the fuzzy similarity measure and compares the performance of the fuzzy similarity measure to similarity computation by taking the weighted sum of the individual attribute similarities. Section 6 concludes this paper and discusses future research directions.

2 A CBR System for Radiotherapy Treatment Planning

This section describes the CBR system under development and the case attributes. The medical physicists at the City Hospital create the treatment plans based on the relative location of the tumour (planning target volume or PTV) and organs at risk (OAR) structures outlined by the oncologist on the patient images. Roentgen [1], which is a CBR system for lung and thorax cancer, uses patient geometric descriptors as case attributes. A similar idea has been implemented in our CBR system. We have identified six geometrical descriptors, which attempt to capture geometric information about the tumour and the spatial relationship between the tumour and OAR. The attributes (shown in Fig.1) are listed below:

1. *Angle, A*: This is the angle, given in degrees from [0,360] interval, between the line connecting the centre of the PTV and the origin of the image coordinate system and the line connecting the centre of the OAR and the origin.
2. *Distance, E*: The distance is defined as the minimum edge to edge distance connecting the outline of the PTV and the OAR and it is given in mm.
3. *Volume, V*: The attribute refers to the volume of the PTV, given in mm^3.
4. *Body – PTV volume ratio, R*: This is the ratio of the PTV volume to the volume of the patient body.
5. *Body – PTV distance, Dt*: This attribute denotes the minimum edge to edge distance in mm between the outline of the PTV and the outline of the body.
6. *PTV – OAR Spatial Relationship, P*: This attribute defines the relative position of the PTV with respect to the OAR. It contains six labels: left, right, posterior, anterior, superior and inferior, which take binary values.

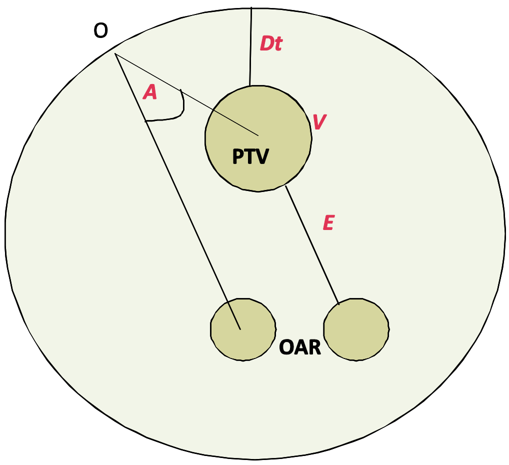

Fig.1.: Case Attributes *A, E, V* and *Dt*

The output of the CBR system consists of the treatment plan parameters. Currently, we consider the number of beams used to irradiate the patient and the beam angles. Each beam angle consists of the sum of the angle of the gantry (which applies the beams to the patient) and the patient couch angle. Fig.2 shows an example of a treatment plan.

Treatment plan of a case	
Gantry Angle (degrees)	Patient Couch Angle (degrees)
30	300
308	0
270	0
166	0

Fig.2.Treatment plan showing four beams and their beam angles

The case base contains patient cases of previously treated brain cancer patients. The cases in the case base consist of the case attributes and the treatment plan parameters. In the developed CBR system, case retrieval is done in two phases. In the first phase, given a target case the similarity between the target case and all cases in the case base is computed. The attributes weights are selected to give more importance to attributes that are relevant to beam number determination. The three most similar cases are retrieved and the mode of the beam numbers suggested by these three cases *BN* is recorded by the system. In the second phase, the case base is filtered so that all cases whose treatment plans have the same beam number *BN* as suggested in phase I are available for retrieval. The attribute weights are selected so as to give more importance to attributes that are relevant to beam angles retrieval. Again, the similarity is computed between the target case and the cases in the case base that have beam number *BN*. The most similar case is retrieved and the beam angles of the treatment plan *BA* are recorded. In the case of beam angles retrieval only one case is retrieved. The beam number *BN* and the beam angles *BA* form the solution of the CBR system for the new case [2]. Experiments showed that retrieving three cases in phase I and retrieving one case in phase II resulted in the best success rate of the system. Retrieval is performed in two phases so that in each phase, attribute weights optimal for beam number of beam angles retrieval can be used. Retrieving the treatment plans sequentially and filtering the case base in phase II according to the number of beams *BN* determined in phase I, ensures that both treatment plans are compatible with each other (in other words, they both have the same number of beams *BN*). The system uses local attribute weights, which vary based on the attribute values of the target case. These weights are applied using rules that have been previously learnt from training data [3].

3 Motivation

In many CBR systems, the aggregate similarity S between two cases C_T and C_C is computed as the weighted sum of the individual attribute similarities as shown in expression (1)

$$S = 1 - \sqrt{\sum_{l=A,E,V,R,Dt,P} \mathrm{w_l}\left(v_{T,l} - v_{C,l}\right)^2} \qquad (1)$$

Where $v_{T,l}$ and $v_{C;l}$ are the values of attributes l, l= A, E, V, R, Dt and P of cases C_T and $C_{C,}$ respectively and w_l denotes the attributes weights. One of the disadvantages of this approach is that it does not take into account during aggregation that the numerical similarity values of attributes might not be comparable. This can be seen by viewing the distribution of attribute similarity values found across the case base. In order to obtain an idea of the distribution of attribute similarities, we calculated the similarity between each case and every other case (with the same OAR) in our case base considering one attribute at a time. The graph on the left of Fig.3 shows the similarity (in terms of the difference in attribute values) calculated using a leave-one-out strategy between each case in the case base consecutively used as the target case and the other cases in the case base. The similarity values are arranged in ascending order. It can be seen that in spite of the distribution of similarity values for each attribute being similar at the extreme points of small and large similarity, the similarity curves differ for each attribute in between. The graph on the right of Fig.3 shows the frequency distribution of the similarities calculated similarly as above for each attribute. Again, it can be seen that though the frequency of larger similarity values increases, in general, for all attributes, the actual similarity values differ substantially between attributes. It can be seen from the graph that the similarity values of all attributes, except *E* and *V*, concentrate towards the higher end of the similarity values spectrum, in particular for attributes *A*, *R* and *Dt.* As an example of the uneven distribution of attribute similarity values, consider the 15th percentile of the similarity values of attribute *R*, which is 0.89. This means that only 15% of similarity values between cases have a similarity of less than 0.89. In contrast, about 93% of the case similarities calculated with respect to attribute E have a value below 0.89. If for instance, the similarity between the target case and a case *A* with respect to case attribute *R*, $S_{AT,R}$ = 0.90 and the similarity between target case and a case *B*, $S_{BT,R}$=0.95, both similarity values would be considered as large. However, since 85% of similarity values with respect to *R* in the case base are more than 0.89, $S_{AT,R}$ is relatively much smaller than $S_{BT,R}$. In other words, at the large frequency end of attribute *R*, the difference in similarity values becomes much more significant. This is irrespective of the attribute weights as weighting affects the entire range of attribute values or attribute similarity values.

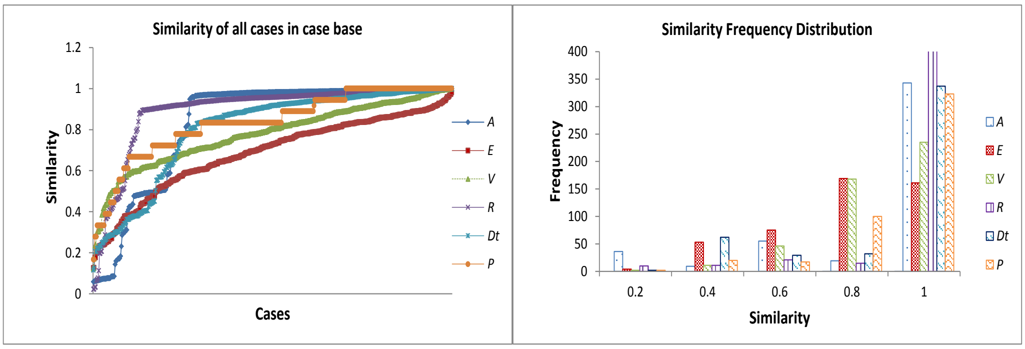

Fig.3: Frequency distribution of similarity values between each case and every other case in the case base for attributes *A, E, V, R, Dt* and *P*.

Based on the contents of the case base, the question, therefore, arises if the numerical similarity values of normalised attributes are actually comparable and can they be summed to provide the total similarity between two cases. In order to sensibly compare attribute similarities and compute the aggregate similarity between them, the CBR system has to understand how to interpret the numerical value of the attribute similarity with respect to the numerical values of the other attribute similarities.

In this work, we investigate how to facilitate the interpretation of numerical similarity values so that the individual attribute similarities are comparable and can be combined into an aggregate similarity. The similarity between individual attributes is interpreted as *Large, Medium* or *Small* depending on how they compare numerically to the similarity between the target case and all cases in the case base with respect to that attribute. We propose the use of fuzzy sets to describe for each attribute what similarity value constitutes a large, medium or small similarity based on the similarity values found in the case base.

4 The Fuzzy, Non-linear Similarity Measure

Fuzzy sets, as opposed to classical sets, allow partial membership of an element to a set [4, 5]. Fuzzy set theory has been widely used in modeling the inference process and in the similarity measure of CBR systems [6-9]. Bonissone et al. [10] suggested that fuzzy set theory is very applicable to CBR since the cases stored in a CBR system are inherently fuzzy in nature as the usefulness of the case solution is normally a matter of degree as evaluated by the similarity measure. Another advantage of using fuzzy sets is that when the attribute values are expressed in terms of membership functions, it eliminates the need for normalisation of attribute values [11]. Fuzzy set theory is also very useful in clinical CBR systems as it can define inexact medical terms. Begum et al. designed a CBR system to classify and diagnose stress in individuals [12].

In this work, we use fuzzy set theory to express the similarity between two cases. To give the total similarity between two cases, individual attribute similarity values have to be aggregated. However, instead of calculating the weighted sum of the attribute similarities as shown in expression (1), we define fuzzy sets *Large*, *Med* and *Small*, which denote large, medium and small similarity, respectively, for each attribute. The fuzzy membership functions of the three sets are defined for each attribute *A*, *E*, *V*, *Dt*, *R* and *P* based on the minimum, maximum and average values of the corresponding similarity values found across the case base for that attribute. They, therefore, give a realistic indication of what constitutes a relatively large, medium or small similarity for an attribute. Given a target case C_T and a case from the case base C_C, the membership degree of the attribute similarity between these two cases to fuzzy sets *Large*, *Med* and *Small* is computed for each attribute *l*, *l = A, E, V, R, Dt, P*. The aggregate similarity consists of the *Large*, *Med* and *Small* component M_s, defined as the sum of the membership degrees of the attribute similarities to their corresponding fuzzy sets *Large*, *Med* and *Small*, as shown in expression (2).

$$M_s = \sum_{l=A,E,V,R,Dt,P} w_l \mu_{l,s} \tag{2}$$

$$S_F = w_{Large} \sum_{l=A,E,V,R,Dt,P} w_l \left(\mu_{l,Large}\right) + w_{Med} \sum_{l=A,E,V,R,Dt,P} w_l \left(\mu_{l,Med}\right) - w_{Small} \sum_{l=A,E,V,R,Dt,P} w_l \left(\mu_{l,Small}\right) \tag{3}$$

where s = *Large*, *Med*, *Small*, w_l denotes the weight of attribute *l*, *l = A, E, V, R, Dt, P*, and $\mu_{l,s}$ is the membership degree of the attribute similarity to the fuzzy sets *Large*, *Med* and *Small.* The terms w_{Large}, w_{Med} and w_{Small} denote the weights of the large, medium and small fuzzy components. A large value of component M_{Large}, M_{Med} and M_{Small} indicates a large, medium and small aggregate similarity between two cases, respectively. That is, M_{Large} has a positive effect on the aggregate similarity between two cases, while M_{Small} has a negative or penalizing effect. M_{Med} either adds to or penalises the aggregate similarity depending on whether the value of w_{Med} is found to be positive or negative. The aggregate similarity S_F between two cases is defined as the net contribution of M_{Large}, M_{Med} and M_{Small} as shown in expression (3).
An added advantage of using expression (3) to compute the similarity measure between two cases is that the similarity and its converse, the dissimilarity can be expressed separately and can therefore be weighted individually. If the total similarity value is given by the sum of the weighted attribute similarity values, the aggregate similarity is always a function of the attribute similarity values. In other words, large attribute similarity values act to increase the aggregate similarity by a large amount while small attribute similarity values also increase the aggregate similarity, but by a smaller amount. That is, no matter how similar or dissimilar two cases are to each other with respect to an attribute, the attribute similarity always contributes positively to the aggregate similarity. An alternative method of case retrieval would be to consider the dissimilarity between cases or to measure the extent by which two cases are different from each other. The weights w_{Large}, $w_{Med,}$ w_{Small} determine the relative contribution of the large, medium and small fuzzy components and are determined in a supervised learning approach using labeled training data (For details, please see [2]). By changing the weights w_{Large}, $w_{Med,}$ w_{Small} we can control the extent to which the similarity and dissimilarity components contribute to the similarity between two cases.

4.1 Fuzzy Membership Functions

The membership function of a fuzzy set assigns to each crisp object of a set a grade of membership to a fuzzy set known as membership degree or membership value [4]. The membership grades usually range from [0,1]. Fuzzy membership functions can be learnt from existing data [13, 14] or defined a priori, usually with the help of domain experts. In this work, the fuzzy membership functions are defined based on the data found in the case base and take the form of triangular membership functions. A triangular membership function is defined by the support and the model point of the triangle. In order to approximately model the distribution of the similarity values among the cases in the case base of an attribute, the left and right support and model point of the triangular membership function are given by the minimum, maximum and average values, respectively, of the similarities found in the case base. In other words, each case of the case base is made the target case in a leave-one-out fashion and the similarity values between the target cases and all other cases in the case base are computed with respect to each attribute. The maximum, minimum and average of the similarity values of each attribute is identified. Expression (4) represents the rules used to assign membership grades μ_{Small}, μ_{Med}, μ_{Large} of attribute similarity S_l to fuzzy sets *Small*, *Med* and *Large*, respectively, where S_{min}, S_{avg} and S_{max} are the minimum, average and maximum values of the similarities found in the training case base between each case and every other case with respect to an attribute. The shapes of the fuzzy membership functions of attributes *A* and *E* are shown in Fig 2.

$$\mu_{Small} = \begin{cases} 1 \; for \; S_l < \mathrm{S_{min}} \\ 0 \; for \; S_l > \mathrm{S_{avg}} \\ \dfrac{\mathrm{S_{avg}} - S_l}{\mathrm{S_{avg}} - \mathrm{S_{min}}} \; for \; \mathrm{S_{min}} \le S_l \le \mathrm{S_{avg}} \end{cases}$$

$$\mu_{Med} = \begin{cases} \dfrac{S_l - \mathrm{S_{min}}}{\mathrm{S_{avg}} - \mathrm{S_{min}}} \; for \; \mathrm{S_{min}} < S_l < \mathrm{S_{avg}} \\ \dfrac{\mathrm{S_{max}} - S_l}{\mathrm{S_{max}} - \mathrm{S_{avg}}} \; for \; S_{avg} < S_l < \mathrm{S_{max}} \\ 0 \; for \; \mathrm{S_{min}} \le S_l \ge \mathrm{S_{max}} \end{cases} \qquad (4)$$

$$\mu_{Large} = \begin{cases} 1 \; for \; S_l > \mathrm{S_{max}} \\ 0 \; for \; S_l < \mathrm{S_{avg}} \\ \dfrac{S_l - S_{avg}}{\mathrm{S_{max}} - \mathrm{S_{avg}}} \; for \; \mathrm{S_{avg}} \le S_l \le \mathrm{S_{max}} \end{cases}$$

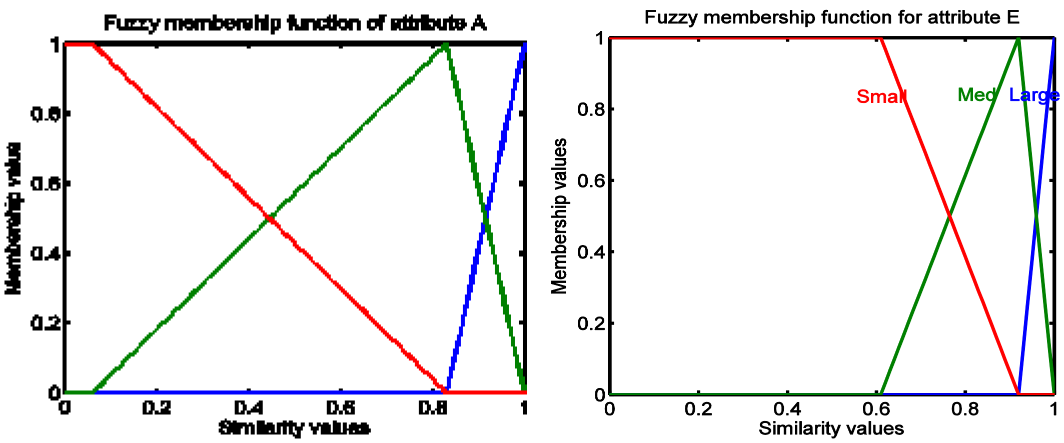

Fig.4: Fuzzy membership functions of attributes *A* and *E* for fuzzy sets *Large*, *Med* and *Small*.

4.2 Local Fuzzy Membership Functions

In the previous sections, the membership functions that were used to assign fuzzy membership grades to the crisp attribute similarity values were global, that is the same membership functions were used for all target cases. In this section, we examine the use of defining the membership functions online (i.e. at the time of retrieval) with respect to the attribute similarities of the target case. To illustrate the rationale behind introducing a new approach to defining membership functions, let us consider two random cases from the case base, C_A and C_B. Fig.5a) and c) show the attribute similarities between C_A and C_B, and all other cases in the case base. For both cases, it can be seen that the similarity values concentrate between 0.9 and 1 with respect to attributes *A*, *E* and *Dt*. However, the attribute similarities with respect to attributes *R* and *P* are very different for cases C_A and C_B. Similarly, graphs c) and d), which present the frequency distribution of the attribute similarity values for cases C_A and C_B show considerable differences.

Comparing Fig.5 with Fig.3, which shows the distribution of attribute similarities over all cases in the case base, it can be seen that the similarity distribution for an individual case is very different. It is likely that the globally defined fuzzy membership functions for fuzzy sets *Large*, *Med* and *Small* are not appropriate for every target case, since the maximum, minimum and the average similarity values of the target case and the cases in the case base show wide variations.
For this reason, we define the membership functions individually for each target case. Given a target case, the CBR system converts the crisp attribute similarity values between the target case and the cases in the case base using the membership function in expression (4). However, the values of S_{min}, S_{max} and S_{avg} are computed online for each target case based on the similarity values between the target case and the cases in the case base available for retrieval. The case with the highest aggregate fuzzy similarity value according to expression (3) is then retrieved to be used in the solution of the target case.

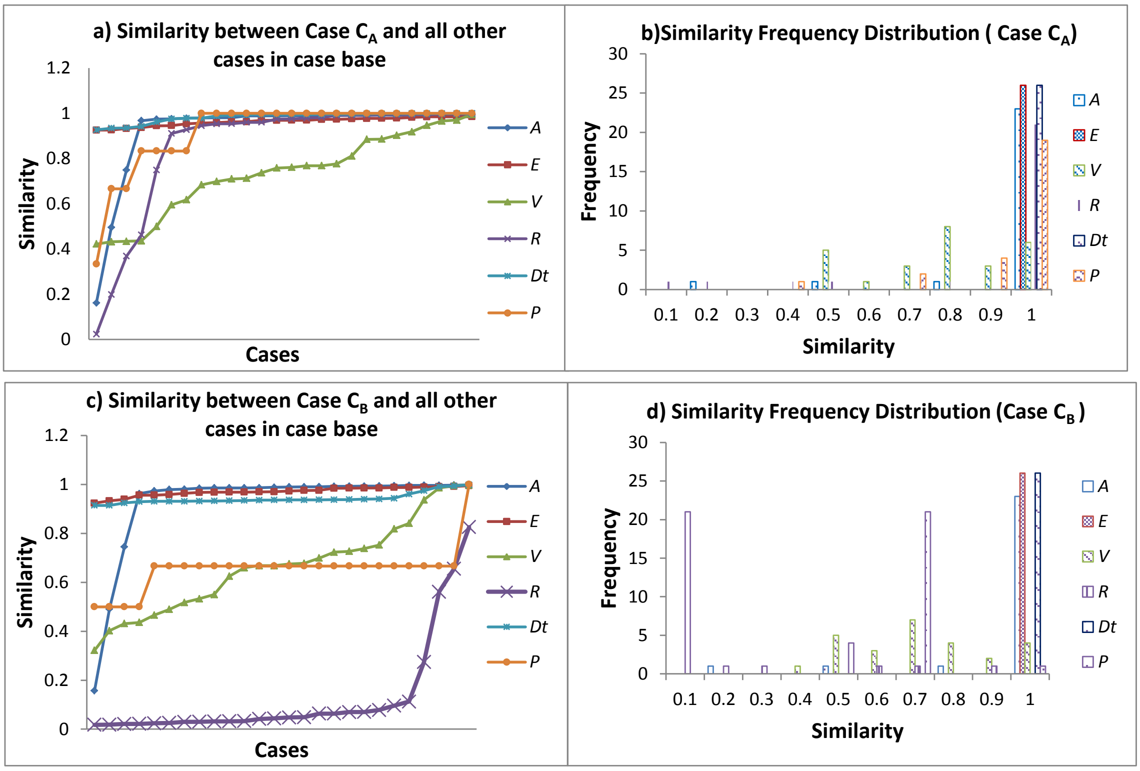

Fig.5: Graph a) and c) show the similarity between two cases C_A and C_B and the other cases in the case base. Graph b) and d) show their frequency distribution

5 Evaluation of Fuzzy, Non-Linear Similarity Measure

In order to test the performance of the fuzzy similarity measure both with global and with local fuzzy membership functions, the retrieval mechanism was evaluated using leave-one-out cross validation with 22 test cases and a case base of 86 cases. (Please note that the parameters of the CBR system, such as the local fuzzy membership functions were determined using 64 training cases, which are different to the 22 test cases used for evaluation.) Each test case was consecutively made the target case. The similarity S_l between attribute values $v_{T,l}$ of the target case and attribute values $v_{C,l}$ of a case in the case base with respect to each attribute *l*, *l*= *A*, *E*, *V R*, *D t*and *P* was computed using expression (5).

$$S_l = 1 - \left(v_{T,l} - v_{v,l}\right) \qquad (5)$$

The attribute similarities between the test target cases and the cases in the case base were fuzzified by converting them to membership grades of fuzzy sets *Large*, *Med* and *Small* using expression (4). The fuzzified attribute similarity values were combined into an aggregate similarity value using the fuzzy, non-linear similarity measure in expression (3). For each target case, two cases along with their treatment plans were retrieved, one to suggest the number of beams and the other to suggest the beam angles. The difference in the number of beams between the first retrieved treatment plan and the known treatment plan of the target case constitutes the beam number retrieval error, E_{BN}. The average difference in the beam angles between the second retrieved treatment plan and the known treatment plan of the target case constitutes the beam angles retrieval error E_{BA}. The values of E_{BN} and E_{BA} were averaged over all 22 test cases.

This process was repeated twice. The first time, the attribute similarity values were assigned membership grades to fuzzy sets *Large*, *Med* and *Small* using globally defined membership functions and the second time, using locally defined membership functions for each individual target case. The performance of the fuzzy similarity measure was also compared with the performance of using the similarity measure shown in expression (1). In each case, the averaged error values of E_{BN} and E_{BA} between the treatment plan of the retrieved cases and the known treatment plan of the target cases were computed. The error values E_{BN} and E_{BAn} obtained using the similarity measure in expression (1), using the fuzzy similarity measure with globally defined membership functions and using the fuzzy similarity measure with locally defined membership functions are shown in Table 1.

Table 1. Beam number error E_{BN} and beam angles error E_{BA} obtained using different similarity measures on the test cases.

	Error	**Success Rate (%)**
E_{BN} using weighted sum of individual attribute values	0.27	77.3
E_{BA} using weighted sum of individual attribute values	22.99°	81.9
E_{BN} using fuzzy similarity measure with globally defined membership functions	0.36	68
E_{BA} using fuzzy similarity measure with globally defined membership functions	22.18°	82
E_{BN} using fuzzy similarity measure with locally defined membership functions	0.23	82
E_{BA} using fuzzy similarity measure with locally defined membership functions	22.76°	82

The success rate refers to the number of target cases, in which retrieval was deemed successful. In the case of beam number retrieval in phase I, retrieval is considered to be successful if the treatment plan of the retrieved case has the same number of beams as the known treatment plan of the target case (i.e. E_{BN}=0). In the case of beam angles retrieval in phase II, retrieval is considered successful if the average difference in the beam angles between the retrieved treatment plan and the known treatment plan of the target case is less than 30˚ (which is the value deemed acceptable by the medical physicists at the City Hospital as an error of 30˚ can be easily corrected without changing the number of beams, either by the user themselves or by a CBR adaptation module). The more stringent definition of a successful beam number retrieval reflects that adaptation of beam number either by the user or by a CBR adaptation module is more difficult than adaptation of the beam angles.

Comparing the retrieval error of the CBR system with respect to the beam number error, E_{BN}, it can be seen that the success rate of the first method of similarity calculation is actually better than the fuzzy similarity measure with globally defined membership functions. However, the success rate considerably improves for beam number retrieval when using the fuzzy similarity measure with locally defined membership functions. This indicates that the fuzzy similarity measure with globally defined membership functions does not model the distribution of attribute similarity values into large, small and medium sets accurately. The fact that the success rate increases considerably when using the fuzzy similarity measure with locally defined weights confirms the importance of accurately interpreting the attribute similarity values in terms of the large, medium and small sets. With respect to the beam angles error, E_{BA} the retrieval error and success rate remain fairly constant irrespective of the similarity measure used. It is possible that the beam angles retrieval is more stable with respect to the distribution of the attribute similarity. This is further confirmed by the fact that using locally defined fuzzy membership functions that model the distribution more accurately does not improve the success rate any further either. The case attributes are weighted locally [3], i.e. the attribute weights of each target case depend on the attribute values of the target case, and the attribute weights are different during beam number and beam angles retrieval it appears that in the case of beam angles retrieval the case attributes with a large weight do not show much variation in similarity values. This might explain why the performance of beam angles retrieval is not affected by the choice of similarity measure.

6 Conclusion

This paper introduced the non-linear fuzzy similarity measure, in particular, the novel concept of using locally defined fuzzy membership functions. The advantages of this similarity measure over the commonly used method of calculating the similarity as the weighted sum of individual attribute similarities are two-fold:

1. It takes into account that the distribution of attribute similarities varies for each attribute and therefore the numerical attribute similarity values cannot be merely summed to give an accurate representation of the true similarity between two cases.
2. By grouping the similarity into large, medium and small similarity components, both similarity and dissimilarity can be expressed and separately weighted.

The fuzzy similarity measure works well if the numerical values of attribute similarity values are not comparable. Using locally defined fuzzy membership functions for each target case, substantially improves the success rate for beam number retrieval, indicating that the distribution of attribute similarity values indeed is an important factor in beam number retrieval. However, beam angles retrieval appears to be more stable to the distribution of attribute similarities. In future work, we are planning to investigate further the reason behind why the beam angles retrieval is not considerably improved using the fuzzy similarity measure and the link between the attribute values, their weights and beam angles retrieval.

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